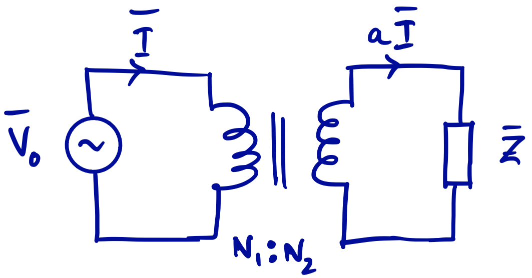
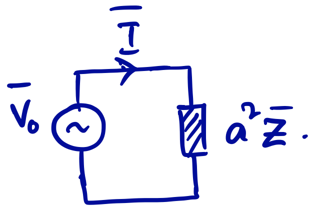
Ideal transformer with sinusoidal voltage source.



$a = N_1 / N_2$ defines the turns ratio.

- $\bar{Z}$ referred to the primary side :



- Computing the max. flux in the ideal transformer:

$$v_0(t) = N_1 \frac{d\phi}{dt} \Rightarrow \phi(t) = \frac{1}{N_1} \int v_0(t)\, dt$$

$$= \frac{1}{N_1} \cdot \int \sqrt{2}\,|\bar{V}_0| \cos\left(\omega t + \angle \bar{V}_0\right) dt$$

$$= \frac{\sqrt{2}\,|\bar{V}_0|}{N_1} \frac{\sin \omega t}{\omega}$$

$$\therefore \phi_{max} = \frac{\sqrt{2}\,|\bar{V}_0|}{N_1\, 2\pi f},$$ where f = frequency (usuall 60 Hz).

Written differently,

$|\bar{V}_0|$ (RMS value of voltage)

$$= \sqrt{2}\,\pi\, N_1 f \phi_{max}.$$

$$= 4.44\, N_1 f \phi_{max}.$$

... important to remember.

An example: A 480 V/120 V transformer supplies a resistive load a power of 9.6 kVA at its rated voltage. What is the resistance of the load referred to the primary side?

New term: "rated voltage"

Usually, all equipments come with a "rating" that describes the maximum, and roughly, the normal level of that quantity for that equipment, much like speed limit on a road.

- 480 V/120 V transformer

  $\Rightarrow N_1/N_2 = 480/120 = 4:1$.

  $\Rightarrow$ voltage across load

  = the rated voltage on the secondary side

  = 120 V.

- Power drawn = 9.6 kVA, Voltage = 120 V.

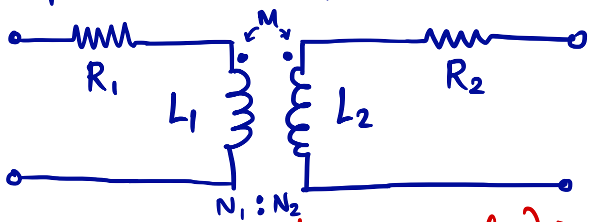
  $$\Rightarrow R = \frac{(120\,V)^2}{9.6\,kVA} = 1.5\,\Omega.$$

- Referred to the primary side, that resistance is $(N_1/N_2)^2 \cdot R = 4^2 \times 1.5\,\Omega$

  $= 24\,\Omega$.

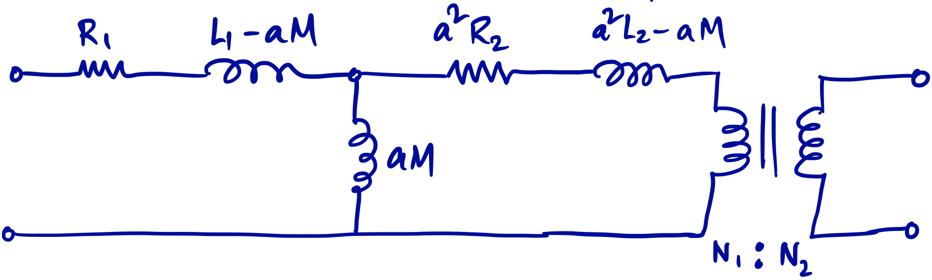
Representing a real transformer using a combination of inductors & an ideal transformer.

- Consider a transformer described by a coupled coil representation:

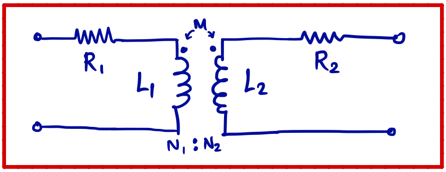


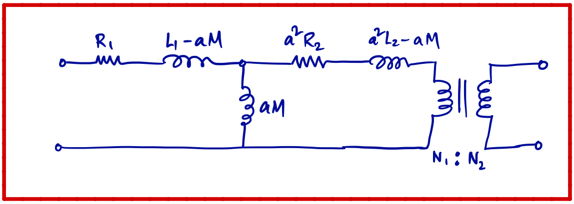
Notice that we have included resistances R_1 & R_2 to denote the resistances in the wires.

- We will show that it is equivalent to



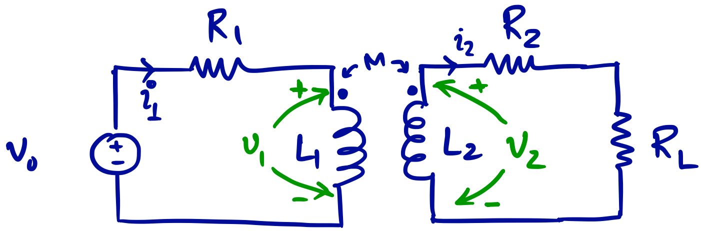
- Agenda : To show these two circuits are eq.





- Method : ① Add a source and a load.
  ② Write down "loop equations" or KVL for both.
  ③ Show they are the same.

Circuit 1 :

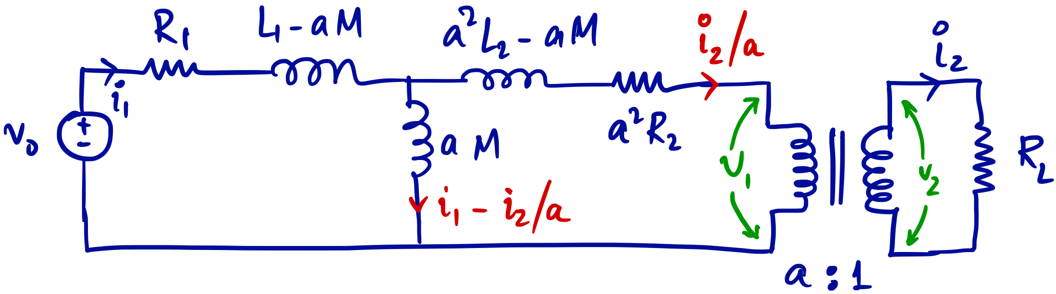


- $v_0 - i_1 R_1 = v_1$
- $v_2 - i_2 R_2 - i_2 R_2 = 0$
- $v_1 = L_1 \frac{d}{dt}(i_1) + M \frac{d}{dt}(-i_2)$
- $v_2 = L_2 \frac{d}{dt}(-i_2) + M \frac{d}{dt}(i_1)$.

Eliminating v_1 & v_2, we get

$$\boxed{\begin{aligned} &\bullet\; v_0 - i_1 R_1 - L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = 0. \\ &\bullet\; -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} - i_2 R_2 - i_2 R_L = 0. \end{aligned}}$$

Circuit 2:



KVL for this circuit:

- $v_0 - i_1 R_1 - (L_1 - aM) \dfrac{di_1}{dt} - (aM) \dfrac{d}{dt}\left(i_1 - \dfrac{i_2}{a}\right) = 0$
- $(aM)\dfrac{d}{dt}\left(i_1 - \dfrac{i_2}{a}\right) - (a^2 L_2 - aM)\dfrac{d}{dt}\left(i_2/a\right) - \dfrac{i_2}{a} \cdot a^2 R_2 = v_1$
- $v_1/v_2 = a.$
- $v_2 = i_2 R_L.$

• Simplify the first eq^n.

$$v_0 - i_1 R_1 - (L_1 - aM)\frac{di_1}{dt} - (aM)\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right) = 0$$

iff $$v_0 - i_1 R_1 - L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} = 0 .$$

• Let's eliminate v_1 & v_2 from the second eq^n.

Notice that $v_1 = a i_2 R_L$. Then, second eq^n becomes

$$(aM)\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right) - (a^2 L_2 - aM)\frac{d}{dt}(i_2/a) - a i_2 R_2 = a i_2 R_L$$

iff $$\frac{di_1}{dt}(aM) + \frac{di_2}{dt}\left(-\cancel{M} - aL_2 + \cancel{M}\right) - a i_2 R_2 - a i_2 R_L = 0$$

Dividing by 'a' throughout, we get

$$M\frac{di_1}{dt} - L_2\frac{di_2}{dt} - i_2 R_2 - i_2 R_L = 0 .$$

∴ Circuit 2 yields

$$\boxed{\begin{aligned} &v_0 - i_1 R_1 - L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = 0. \\ &M \frac{di_1}{dt} - L_2 \frac{di_2}{dt} - i_2 R_2 - i_2 R_L = 0. \end{aligned}}$$

Compare it with that derived from circuit 1, reproduced below:

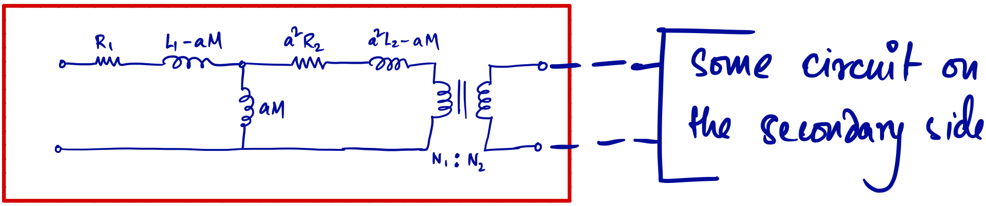
$$\boxed{\begin{aligned} &\circ \; v_0 - i_1 R_1 - L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = 0. \\ &\circ \; -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} - i_2 R_2 - i_2 R_L = 0. \end{aligned}}$$

They match!

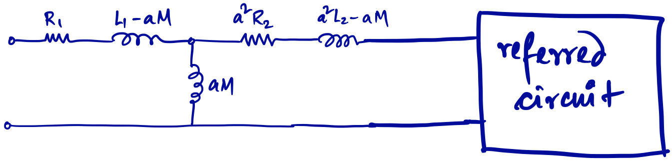
Therefore, they are equivalent circuits.

- A real transformer therefore admits two equivalent representations :-
  - A coupled coil representation
  - An ideal transformer + inductor representation.

Q. Is there any advantage of the ideal transformer + inductor representation ?



Can be referred to the primary side.



Transformer disappears ⇒ simplifies analysis.